

# UMTYMP CALCULUS: A SPECIAL OPPORTUNITY

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*This is an expanded version of a presentation given to UMTYMP Math Analysis parents and students about how UMTYMP Calculus compares to other options. It is not meant to be a comprehensive overview of the program, so please contact Professor Rogness or the MathCEP office if you have any additional questions or would like more details.*

## INTRODUCTION

Thirty years ago, UMTYMP was essentially the only way for local high school students to take Calculus, so it was assumed that any UMTYMP student who finished the high school component would continue on to the Calculus component. Today's Math Analysis students have a wide variety of options, however: most high schools offer some form of Advanced Placement (AP) Calculus courses, and International Baccalaureate (IB) classes are common as well. Some schools even offer College in the Schools classes in which the students simultaneously earn high school and University credit for one semester of Calculus. Students in grades 11 or 12 can also directly enroll in college courses through the Post Secondary Enrollment Options (PSEO) program, in which the state of Minnesota covers the cost of tuition, fees and textbooks.

Even with all the other options, there are still good reasons to continue with our Calculus courses. If UMTYMP were simply a standard calculus course, it would have disappeared many years ago because it no longer would have been needed. Instead, UMTYMP is still thriving because it is most definitely not a standard calculus sequence, and offers many advantages over the alternatives.

Before continuing, it is worth pointing out that UMTYMP is not (and never has been) intended as a program to teach high school level mathematics and then send students back to their own schools for the rest of their mathematical education. The University of Minnesota exists to serve the people of our great state, but teaching high school courses is outside of our primary mission. Indeed, the high school UMTYMP program exists for the sole purpose of preparing students for UMTYMP Calculus. When we admit students to UMTYMP Algebra, we expect and hope that they will stay for the entire five year program.

This goal of this presentation is to provide you with enough information to help you decide what is best for you. I can speak from personal experience about the various options. As a high school student, I took calculus as a junior at St. Paul Central High School. As a faculty member in the school of mathematics, I am the coordinator for our University's College in the Schools (CIS) Calculus course, and I visit 6-10 high school classrooms per year. I have also taught in the UMTYMP Calculus sequence for over ten years and have an intimate knowledge of the program.

## HIGH SCHOOL, PSEO AND CIS

Math Analysis students have many options in front of them. This section runs through common choices and describes some of the pros and cons of each. As the Director of UMTYMP I tend to think everybody should continue in UMTYMP if possible, but it is worth repeating that I was not an UMTYMP student myself. As a high school student I took all of my mathematics courses (including Calculus and beyond) at my regular high school in St. Paul, and it served me well. **Hence it is absolutely not my desire or intent to denigrate any local high school courses.** The Twin Cities area is lucky to have many superb high school mathematics teachers and students are fortunate to be in their classrooms. (In fact, I still think so highly of my high school teachers that we hired one of them to teach in the UMTYMP high school component.)

**Staying at one's High School.** This option is cheaper, involves less travel, and allows students to take courses with friends at their own schools. Students may run out of mathematics courses.

This is the option I took as a high school student. I learned a lot and eventually earned a Ph.D. in Mathematics. However, I took Calculus as a junior. If I had been more advanced and taken Calculus earlier I would have run out of classes to take at my high school. This is the case for virtually every UMTYMP Math Analysis student who returns to their regular school, which brings us to the next option.

**High School followed by PSEO.** After taking a year or two of high school courses, students can enroll in 2000 level courses through PSEO. This option is still cheaper, and students earn credits on a college transcript.

*Potential Downsides of PSEO vs. UMTYMP.*

- Larger class sizes with a general population of students. Some 2000 level courses may have over 150 students in the lecture, many

of whom are there because of a graduation requirement instead of a deep interest in mathematics.

- Difficult schedules. PSEO students are registered in regular University courses which might run, say, MWF 10:10-11:00. This can wreak havoc for a student still taking courses at their high school.
- Instructors who are not accustomed to high school students and will not offer any extra support for younger students. Office hours, like classes themselves, do not mesh well with a high school student's schedule.
- No sense of community. Each course may have a totally different group of students, making it difficult to form friendships and study groups from class to class.

**College in the Schools.** Some high schools offer *College in the Schools* (CIS) or *dual enrollment* Calculus courses. The teachers in these courses must be approved by a college or university; students receive credit on their high school transcript and on a transcript at the college or university. (Hence the name: students are simultaneously enrolled in a high school level and college level course.) If your school offers such a class, you should check if the CIS program at the sponsoring college or university is accredited, which among other things means a faculty member is carefully monitoring the high school teachers' syllabi and writing all of the exams. Non-accredited dual enrollment courses can still be very good, but it can be more difficult to receive transfer credit and placement for them at other colleges.

The University of Minnesota has an accredited CIS program which offers Calculus I at more than a dozen high schools in and around the Twin Cities metro area. As it happens, I am the faculty coordinator for this CIS course, so I can speak with some authority on the differences between CIS and UMTYMP. CIS calculus is a great option for students who want to stay at their own school during the day, but you should be aware of the following facts:

- At the end of the year, students only receive credit for one semester of Calculus, Math 1371, not two.
- Students prepared for UMTYMP might not be challenged by CIS Calculus. In addition to the slower pace, the material is not covered as deeply. As a simple comparison, the hardest problems I write for CIS Calculus exams are comparable to the easiest problems on UMTYMP exams. This is not to say that UMTYMP Calculus exams are horribly difficult, but rather that UMTYMP is simply a higher level mathematics course than Math 1371.

## THE UMTYMP OPTION

Broadly speaking, there are two aspects of UMTYMP which differ from the other options above: the course content (especially in Calculus II and III) and the mathematical culture.

**Course Content.** One of the biggest differences between UMTYMP and the other options is the curriculum, both in terms of topics and the depth at which they are taught. To make this difference very concrete, consider how UMTYMP is viewed in our own department. We have four different sequences encompassing single variable Calculus through Linear Algebra and Multivariable Calculus: for liberal arts students, for science and engineering students, for Honors College students, and UMTYMP. Of the four, our department internally considers UMTYMP to be the most rigorous. Students who finish UMTYMP Calculus III are excused from 3000-level coursework which is required of all other students (including Honors College students) before taking upper division courses.

By comparison, students who have passed AP/IB exams are given credit and/or placement in the liberal arts or science and engineering sequences; after completing their sequence they continue on to the 3000 level course.

Our Calculus I course actually looks quite similar to any AP/IB or college Calculus course in terms of topics covered. (The mathematical culture and writing expectations are very different; see below.) Our Linear Algebra and Multivariable Calculus courses are at a different level, however. Most Linear Algebra (or Linear Algebra and Differential Equations) courses are very heavy on matrix algebra and other calculations, whereas ours takes a geometric, theoretical approach. Once we have covered Linear Algebra, we can do Multivariable Calculus in full generality in Calculus III; the vast majority of Multivariable Calculus courses taught at colleges or high schools use the last third of a large Calculus text, which does not assume any Linear Algebra and therefore cannot cover all of the relevant definitions and ideas. We use a Vector Calculus book which assumes knowledge of matrices and linear transformations and can therefore "tell the whole story." There is nothing inherently wrong with using a standard Calculus book for Multivariable—in fact, that's why my own Multivariable class was like as a student—but anybody who is interested in mathematics, physics or certain types of engineering will eventually have to learn the more general version, so there is an advantage to doing it that way the first time.

For those with a mathematical background, the following list gives a short summary of the curriculum through the six-semester UMTYMP Calculus sequence. If this is outside of your comfort zone and

	UMTYMP C1 Fall	UMTYMP C1 Spring	UMTYMP C2 Fall	UMTYMP C2 Spring	UMTYMP C3 Fall	UMTYMP C3 Spring
<b>Advanced Placement</b>	AP Calculus AB					
	AP Calculus BC					
<b>Local HS</b>	AP Calculus BC					Multivariable
<b>CIS</b>	Math 1371					
<b>UMN</b>	Math 1271-1272			(Math 2243)		Math 2263
<b>UMN (CSE)</b>	Math 1371-1372			(Math 2373)		Math 2374
<b>UMN (Honors)</b>	Math 1571-15672H		Math 2574H			Math 2573H

TABLE 1. Content comparison between the UMTYMP Calculus Sequence and other options available at high schools at the University.

would make your eyes glaze over, please feel free to skip over the list!

*Calculus I Fall.* Limits, derivatives, applications and geometric interpretation of derivatives. (Extended) Mean Value Theorem.

*Calculus I Spring.* Riemann integral, integration techniques and applications, rigorous treatment of sequences and series.

*Calculus II Fall.* Differential equations. 3D coordinate systems. Vectors and products. Set theory, logic and methods of proof. Precise definitions of limits and proofs of convergence and limit laws.

*Calculus II Spring.* Theoretical linear algebra.

*Calculus III Fall.* Differential geometry of curves and surfaces in  $\mathbb{R}^3$ . Multivariable differentiation and optimization using matrices. Limit definition of multivariable derivative. Taylor’s Theorem in higher dimensions.

*Calculus III Spring.* Topology of  $\mathbb{R}^n$ , vector fields, classical vector analysis (Stokes Theorem, etc.).

*Advanced Topics.* Typically we schedule an Advanced Topics course each year Advanced Topics course for students who have finished UMTYMP but are still in high school. The course emphasizes proof reading and writing. The topics vary but have included complex analysis, abstract algebra, topology, and combinatorics in recent years.

Table ?? gives a visual comparison of the content of the UMTYMP Calculus sequence with some of the other options. *Advanced Placement* refers to the typical AP courses available at most high schools. *Local HS* shows a sequence at a local suburban high school which includes multivariable calculus in the second

year, although no college credit is granted for it. The *CIS* line shows the credit earned in a full year College in the Schools course sponsored by the University of Minnesota. The first two *UMN* lines show the standard and engineering Calculus tracks at the University of Minnesota. The Calculus courses at other local colleges and universities are generally very similar to one of these.

The last line shows an honors level sequence in our department, which is the only sequence comparable to UMTYMP in terms of credits granted and topics covered (albeit in a slightly different order).

The comparisons in Table ?? are somewhat technical. For those with a mathematical background, I’ve included more details in an appendix on page ??.

**Mathematical Culture.** Most math students have little idea of what a professional mathematician does, which is not the situation in other fields. Professor Jill Dietz of St. Olaf College summarized it well at a recent conference, pointing out that students who major in, say, Chemistry, spend years working in labs, learning research procedures and how to use the equipment; by the end of their undergraduate careers they have developed the skills to assist their professors in research or to work on their own projects. In mathematics, Prof. Dietz continued, students are taught a collection of theorems and computational techniques, but all too often it stops there. That is unfortunate, because mathematicians do much more than simply carry out long calculations for the fun of it. We spend our time attacking and solving open-ended problems and communicating our work to others.

One of our goals in UMTYMP is to address this problem by incorporating more open-ended problem solving and projects, and by training our students in mathematical writing, which is a important skill and requires a lot of practice All of our students in the

Calculus sequence write so-called “professional problems,” which are graded not only for mathematical correctness but also the quality of mathematical writing. This is a fairly unique aspect of our program; it is rare for Calculus I students to have a PhD-holding practicing research mathematician grade their mathematical writing and give suggestions about organization of their work, or the subtle differences in wording.

Our aim is that any student who finishes the UMTYMP sequence has enough experience in problem solving and mathematical communication that they are prepared to work on open ended projects, should they so desire. Most years around 5-10 UMTYMP students and graduates have worked on a number of projects with UMTYMP faculty, resulting in a publication in the prestigious Rose-Hulman Undergraduate Mathematics Journal and an upcoming conference presentation. We cannot guarantee that if you continue in UMTYMP you will eventually take part in a research project, have a publication, etc. – 10 students is a small fraction of our overall student population! – but we will continue to work on expanding these opportunities for interested students.

**Advantages of UMTYMP Calculus.** Aside from the curriculum and culture, there are a number of other advantages to the UMTYMP Calculus sequence. I should first explain how the courses are structured. Our Calculus courses are not separated into different classrooms which never interact. Each course has a common lecture where everybody is together in one room. Then students split into workshops of 14-20 students which are led by University faculty or advanced graduate students, many of whom are UMTYMP alumni themselves. In workshop people work together in groups of 3-4 students on a series of problems designed to reinforce or extend the ideas from lecture.

#### *Academic Advantages of UMTYMP.*

- Learn more mathematics, and at a more abstract, theoretical and geometric level.
- Form a tightly knit mathematical community of friends from around the Twin Cities.
- Extensive support system specifically for UMTYMP students.
- Excellent teachers who get to know you over the course of three years.
- Contiguous curriculum.

The last point refers to the fact that each course begins where the previous one left off, and we know exactly what was covered the previous year. That’s not always the case in college level courses. When I teach Multivariable Calculus for the department, for example, my students have take a variety of courses to satisfy the prerequisites for the class. I don’t always know what their previous instructors did, or if

they were pressed for time and skipped something important. In UMTYMP I probably *was* their previous instructor. If not, I can walk down the hall and talk to the UMTYMP staff member who taught the material and find out exactly what examples, ideas, and theorems were covered.

#### *Side benefits of UMTYMP.*

- After finishing UMTYMP, students are routinely admitted into PSEO and go straight to advanced 5000 level courses which have 25 students, not 150.
- Students earn University of Minnesota credits
- UMTYMP is highly valued by colleges and universities.
- Students can get a letter of recommendation for college applications from a University faculty member who has known them and taught them for years.
- There can be extensive opportunities for UMTYMP alumni at the University of Minnesota.

#### STUDENT COMMENTS

I asked students who are in or recently finished UMTYMP Calculus III for their comments on UMTYMP and what they would say to students in Math Analysis who are contemplating their options. At this point in the presentation I was going to share a number of these comments, but received a letter from an alumna who summarized things nicely. Hence I will simply quote her thoughts.

I would highly recommend students to continue on with UMTYMP for several reasons. I would say this program is on an entirely different level than other math courses that students may consider. It offers college level math but teaches it in a setting much more suitable for high school students; this makes students feel more comfortable participating and asking questions, and it allows them to get the most out of the experience.

Importantly, it is taught by teachers who really enjoy both the material and teaching, and this is what really makes a difference in how much a student will enjoy the material. The math course also does an excellent job preparing students for college—it prepares them for a heavier course load, allows them to work in a more independent setting, but also teaches students how to work together and form study groups.

One of the best parts of the course for me was definitely the material. We covered topics that other math courses will

certainly not cover, and I really enjoyed the challenge of learning this material and going beyond the basics.

UMTYMP did an excellent job preparing me for math courses at Harvard, and I was able to get much more out of the math courses that I took because my background was so strong. Overall, this course was a great experience for me.

### FINAL THOUGHTS

Taking UMTYMP Calculus is a big commitment for both students and families, and is not something to be undertaken lightly. However, the last 30 years have shown that it is a worthwhile investment of time and effort, and we hope you consider it seriously. Please feel free to contact our office with any questions.

### APPENDIX

This appendix gives a few more details about the comparisons in Table ?? . It's not intended for the general reader—some parts may only make sense to other math teachers—so I apologize in advance if it makes your head hurt.

Comparing various Calculus sequences turns out to be more difficult than you might expect. The normal three semester sequence at a typical American college or university covers all the material in a huge Calculus textbook – usually about 15 chapters and 1,000 pages, give or take. This is what the “Local HS” option in Table ?? does, as well as the Math 1271/1272/2263 sequence at the University. This sequence generally contains no linear algebra, although there is a linear algebra class (Math 2243) which a student could take in our department. Readers with a science, engineering or mathematical background may understand what I mean when I say that Math 2243 focuses more on matrix computations than on the theory behind vector spaces or linear transformations. UMTYMP takes the latter approach, so 2243 isn't really comparable to the spring semester of UMTYMP Calculus II.

The next highest sequence at the University, Math 1371/1372/2374, takes a slightly different approach. The content of the first year is similar, but Math 2374 includes just enough about matrices and linear transformations to do multivariable calculus with matrices, including the derivative as a linear transformation represented by the total derivative matrix. (Without matrices the courses mentioned above are limited to special two- and three-dimensional cases of multivariable calculus.) However, to cover the extra material, Math 2374 leaves out curvature, which is usually covered in a standard multivariable course. Finally, this sequence also includes Math 2373, which covers a fair

amount of differential equations but very little theoretical linear algebra.

UMTYMP Calculus and the honors level courses in Table ?? cover all of these topics, and generally at a more rigorous level. That means we not only state ideas and theorems, but prove them. We also present “pathological” examples – bizarre mathematical functions which exhibit strange behavior and show why the assumptions and hypotheses in our theorems are important. (UMTYMP students are usually fascinated by those examples.)

If you are trying to compare the UMTYMP Calculus sequence to other options, there are three facets to consider:

**Content.** All Calculus sequences cover a standard list of topics, such as the limit definition of a derivative, integration by parts, sequences and series, and partial derivatives. A short list of topics covered in UMTYMP which may not appear in other options includes:  $\varepsilon$ - $\delta$  definition of limits; Extended Mean Value Theorem; high level treatment of sequences and series including  $\varepsilon$ - $N$  definition and proofs of limit laws; set theory, logical notation and methods of proof; geometric and axiomatic approach to linear algebra, including abstract vector spaces of both finite and infinite dimensions; isomorphisms of vector spaces; discrete and continuous dynamical systems; quadratic forms and Sylvester's Theorem, definite and semi-definite matrices (and the quadratic forms they represent); the total derivative matrix; the multivariable limit definition of the derivative as a linear transformation; the chain rule as the product of matrices; the Hessian matrix; the multivariable Taylor's Theorem for approximating functions; optimization where the second derivative test is described in terms of the determinant of the Hessian; optimization in dimensions above 2 and 3; and the topology of  $\mathbb{R}^n$  (not just the plane).

**Depth.** As mentioned above, it is not enough to simply compare a list of topics. When comparing to other options, it is important to consider the depth at which topics are covered. Are results simply stated and used? Or are they explored, explained, and proved? UMTYMP Calculus courses are treated as honors level courses within our department, and as such we cover topics more rigorously.

**Course Names.** At most colleges and Universities, Calculus I, II and III refer to semester courses. The first two courses cover single variable calculus, and the third semester covers multivariable calculus (without using linear algebra). In UMTYMP, Calculus I, II and III refer to the three *year*-long courses, and cover significantly more material than just single and multivariable calculus.

This often causes confusion – some teachers and counselors are aware that UMTYMP's courses are called Calculus I, II and III, but not that the content doesn't perfectly match the standard usage of those names. Hence rather than comparing course titles, it's worth talking about specific content.

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